

# NASA TECH BRIEF



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## Optimum Multi-Impulse Rendezvous Program

### The problem:

To determine the position vector and time of intermediate impulses so that the total velocity impulse for a resulting rendezvous trajectory is locally minimized.

### The solution:

The Optimum Multi-Impulse Rendezvous (OMIR) Program which determines optimal  $n$ -impulse rendezvous trajectories under the restrictions of two-body motion in free space.

### How it's done:

Given the initial and final states of position and velocity and the transfer time, the OMIR Program determines the number, magnitude and direction, location, and time of intermediate impulses so that the total velocity impulse for the resulting rendezvous trajectory is locally minimized. The method applies Lawden's Primer Vector theory to determine the optimum number of midcourse impulse applications. The primer Vector theory results in four conditions which an impulsive rendezvous trajectory must satisfy for the total velocity impulse required on this trajectory to be locally minimized. The theory is applied to nonoptimum trajectories to develop a method for converging from a nonoptimum reference trajectory which satisfies the given end conditions to a trajectory which satisfies the specified end conditions and Lawden's four optimality conditions. The main result is that the gradient of the coast function, the total velocity impulse, can be expressed as a function of the primer vector derivative discontinuities and the discontinuities in the Hamiltonian at intermediate impulse points in the trajectory. If the number of impulses is fixed, the classical optimality conditions—the Lagrange Multipliers, their derivatives, and the Hamiltonian remain continuous across the trajectory—can all be satisfied by driv-

ing the gradient to zero using an appropriate multi-variable search technique. A modified descent method was found to possess adequate search capabilities for this class of problems. The desirability of adding another impulse after driving the gradient to zero is investigated by testing the resulting Primer Vector history. If Lawden's conditions are violated, another impulse is added and convergence continues; if not, a local optimal trajectory has been achieved.

The optimum  $n$ -impulse trajectory is obtained by improving, in an iterative fashion, a reference trajectory which satisfies the specified problem boundary conditions but may be chosen arbitrarily. The analysis and results presented are for fixed-time rendezvous problems which consist of transferring from one state of position, velocity, and time to another state. Thus, all trajectories encountered in the iterative convergence procedure must satisfy 14 specified boundary conditions. In addition, allowable trajectories are required to be continuous in position coordinates; velocity discontinuities occur at impulse points.

Global optimality is not guaranteed by the methods employed in the OMIR Program. Provisions have been made to search for different extremal solutions, and some success has been achieved with this logic.

### Notes:

1. The program was written in Fortran V language for use on the UNIVAC 1108.
2. Inquiries may be directed to:

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(continued overleaf)

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